for the same example described in the previous paragraph, an additional threefold reduction in CPU time was achieved.

IV. Conclusion

The Iskander–Tumeh method of analysis has been demonstrated to yield accurate results with a much less CPU time. Use of the translational symmetry property to further improve its computational efficiency are also demonstated. Additional saving in CPU time is possible if the near-field nature of the problem is taken into account. Numerical experience suggest that $\Delta z' = 1$ mm, $\Delta z \geq 1$ mm, and a 1% field convergence rate should produce accurate SAR with adequate spatial resolution. Since only the symmetry property associated with F_R and F_θ terms are exploited, this algorithm is also applicable to nonuniformly insulated IDA's, for which one needs only replace (1) with appropriate section-dependent current distributions [7].

Acknowledgment

Dr. J. P. Casey's help in providing the computer program used to compute Casey and Bansal's data shown in Fig. 4 is greatly appreciated.

References

[1] R. W. P. King, B. S. Trembly, and J. Strobehn, "The electromagnetic field of an insulated antenna in a conducting or dielectric medium," *IEEE Trans. Microwave Theory Tech.*, vol. 31, pp. 574–583, July 1983.
[2] J. P. Casey and R. Bansal, "The near field of an insulated dipole in a dissipative dielectric medium," *IEEE Trans. Microwave Theory Tech.*, vol. 34, pp. 459–463, Apr. 1986.
[3] J. C. Camart, J. J. Fabre, B. Prevost, J. Pribetich, and M. Chive, "Coaxial antenna array for 915 MHz interstitial hyperthermia: Design and modelization-Power deposition and heating pattern-Phased array," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 2243–2250, Dec. 1992.
[4] Y. Zhang, N. V. Dubal, R. Takemoto-Hambleton, and W. T. Joines, "The determination of the electromagnetic field and SAR pattern of an interstitial applicator in a dissipative dielectric medium," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1438–1444, Oct. 1988.
[5] Y. Zhang, W. T. Joines, and J. R. Oleson, "Microwave hyperthermia induced by a phased interstitial antenna array," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 217–221, Feb. 1990.
[6] K.L. Clibbon and A. McCowen, "Efficient computation of SAR distributions from interstitial microwave antenna arrays," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 595–600, Apr. 1994.
[7] M. F. Iskander and A. M. Tumeh, "Design optimization of interstitial antennas," *IEEE Trans. Biomed. Eng.*, vol. 36, pp. 238–246, Feb. 1989.
[8] R. F. Harrington, *Time Harmonic Electromagnetic Fields.* New York: McGraw-Hill, 1961, p. 79.

A Fast Integral Equation Technique for Shielded Planar Circuits Defined on Nonuniform Meshes

George V. Eleftheriades, Juan R. Mosig, and Marco Guglielmi

***Abstract*— In this contribution, the groundwork is laid out for the realization of efficient integral-equation/moment-method techniques, with arbitrary types of basis functions, for the computer-aided design (CAD) of geometrically complex packaged microwave and millimeter-wave integrated circuits (MMIC's). The proposed methodology is based on an accelerated evaluation of the Green's functions in a shielded rectangular cavity. Since the acceleration procedure is introduced at the Green's function level, it becomes possible to construct efficient shielded moment method techniques with arbitrary types of basis-functions. As an example, a Method of Moments (MoM) is implemented based on the mixed potential integral equation formulation with a rectangular, but nonuniform and nonfixed, mesh. The entire procedure can be extended to multilayer substrates.**

I. Introduction

In the framework of the Method of Moments (MoM) for shielded circuits, a major component of the CPU time is attributed to filling the MoM matrix due to the large number of summation terms involved [1]–[6]. To date, the most successful technique for addressing this filling problem is by using the fast Fourier transform (FFT) [2]–[4]. Unfortunately, the FFT restricts the underlying discretization to a fixed rectangular mesh with the corresponding subsection size limited to an integral multiple of the basic cell size. For these reasons, the FFT imposes restrictions to the accurate description of the geometries to be analyzed. In addition, the basic cells size, and thus the order of the FFT, are determined by the finest geometrical feature in the circuit and this cannot always be the most efficient choice.

Herein, the groundwork is laid out for the realization of *efficient* moment methods in a shielded environment with *arbitrary* types of basis functions. This becomes possible due to the introduction of a fast scheme for evaluating the Green's functions in a rectangular cavity. The technique begins by extracting the asymptotic part from the usual two-dimensional (2-D) modal summation form of the box Green's function [4], [5]. The asymptotic part depends on the frequency in a trivial manner and thus is expressed in terms of frequency-independent summations. Subsequently, these frequency-independent summations are transformed into a form that involves the exponentially decaying Bessel functions of the second kind. This enables to effectively collapse the original frequency-independent 2-D sinusoidal series into one-dimensional (1-D) ones. Because the acceleration process is applied at the Green's function level, the door opens to the realization of efficient MoM-based techniques with arbitrary types of basis functions.

As an example, a particular moment method has been implemented based on the mixed potential integral equation (MPIE) formulation and a nonuniform/nonfixed rectangular mesh [5], [8]. At the MoM level, special care is taken so that the interaction integrals involving the modified Bessel functions are carried out in an optimum way. Recently, an independent attempt was made in [6] to also accelerate

Manuscript received November 10, 1995; revised August 26, 1996.
G. V. Eleftheriades and J. R. Mosig are with the Laboratoire d' Electromagnetisme et d' Acoustique, Ecole Polytechnique Fédérale de Lausanne, Lausanne, CH-1015, Switzerland.
M. Guglielmi is with the European Space Research Technology Center, European Space Agency, 2200 AG Noordwijk, The Netherlands.
Publisher Item Identifier S 0018-9480(96)08496-7.

the evaluation of the asymptotic MoM matrix elements. In that effort, however, acceleration techniques have been applied directly to the elements of a particular MoM implementation. Therefore, the corresponding results cannot be generalized to arbitrary types of basis functions.

II. Summary of the Formulation

The general form of the structures considered in this article consist of passive microstrip circuits printed on an isotropic dielectric substrate (ε_1, μ_1) of thickness, t, enclosed in a rectangular shielding cavity of dimensions $a \times b \times c$. The corresponding vector and scalar potential Green's functions for $\hat{x}$-directed currents are given by the expressions

$$G_A^{xx} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{j e_{0m} e_{0n} V_{mn}^{\mathrm{TE}}(\omega)}{ab\omega} \cos\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{n\pi y'}{b}\right) \tag{1}$$

$$G_V = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{4j\omega}{abK_{mn}^2}[V_{mn}^{\mathrm{TE}}(\omega) - V_{mn}^{\mathrm{TM}}(\omega)] \sin\left(\frac{m\pi x'}{a}\right) \cdot \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{n\pi y'}{b}\right) \tag{2}$$

where the coefficients $V_{mn}^{\mathrm{TE}}(\omega), V_{mn}^{\mathrm{TM}}(\omega)$ are voltages on equivalent transmission lines corresponding to each $\mathrm{TE}_z/\mathrm{TM}_z$ waveguide mode and describing the longitudinal z-variation of the fields [9]. Also, $K_{mn} \doteq \sqrt{(m\pi/a)^2 + (n\pi/b)^2}$ is the transverse wavenumber and $e_{0m} = 1$ if $m = 0$, otherwise $e_{0m} = 2$. In order to accelerate the computation of the double summations in (1) and (2), we introduce the frequency-independent components of the potential Green's function $\hat{G}_A^{xx,\mathrm{TE}}, \hat{G}_V^{\mathrm{TE}}, \hat{G}_V^{\mathrm{TM}}$ [see (4)–(8), below]. These components can be obtained from the original expressions (1) and (2) by computing the asymptotic values of the voltage coefficients $V_{mn}^{\mathrm{TE}}(\omega), V_{mn}^{\mathrm{TM}}(\omega)$ as the indexes m, n tend to infinity. Now, the frequency-independent components can be added and subtracted to (1) and (2) and the final result is shown in (3)

$$\begin{aligned} G_A^{xx} &= G_{AD}^{xx} + \hat{G}_A^{xx,\mathrm{TE}}, \\ G_V &= G_{VD} - \omega^2 \hat{G}_V^{\mathrm{TE}} + \hat{G}_V^{\mathrm{TM}}. \end{aligned} \tag{3}$$

The entire process of extracting the asymptotic components of the original series (1) and (2) to obtain the rapidly convergent "residual" series G_{AD}^{xx} and G_{VD} can be identified with the well-known Kummer's method of series acceleration [10]. What remains to be done for completing the effort of accelerating the original Green's functions is to also enhance the convergence of the remainder frequency-independent components $\hat{G}_A^{xx,\mathrm{TE}}, \hat{G}_V^{\mathrm{TE}}, \hat{G}_V^{\mathrm{TM}}$

$$\hat{G}_A^{xx\ \mathrm{TE}} = \sum_{m=0}^{\infty} \frac{-\mu_f e_{0m}}{2ab} \cos\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{m\pi x}{a}\right) \cdot S_m^{(1)}(y, y') \tag{4}$$

$$\hat{G}_V^{\mathrm{TE}} = \sum_{m=1}^{\infty} \frac{\mu_f}{ab} \sin\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{m\pi x}{a}\right) \cdot S_m^{(2)}(y, y') \tag{5}$$

$$\hat{G}_V^{\mathrm{TM}} = \sum_{m=1}^{\infty} \frac{1}{ab\varepsilon_f} \sin\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{m\pi x}{a}\right) \cdot S_m^{(1)}(y, y') \tag{6}$$

$$S_m^{(1)}(y, y') = \sum_{n=1}^{\infty} \frac{4\sin(n\pi y'/b)\sin(n\pi y/b)}{K_{mn}} \tag{7}$$

$$S_m^{(2)}(y, y') = \sum_{n=1}^{\infty} \frac{4\sin(n\pi y'/b)\sin(n\pi y/b)}{K_{mn}^3}. \tag{8}$$

In (4)–(6) $\varepsilon_f = 1/\varepsilon_1 + 1/\varepsilon_2, 1/\mu_f = 1/\mu_1 + 1/\mu_2$ with $(\varepsilon_1, \mu_1), (\varepsilon_2, \mu_2)$ being the permittivity and permeability of the two layers adjacent to the current-carrying interface $z = 0$. Using Poisson's summation formula together with Sommerfeld's identity [11] enables to convert the transverse sinusoidal summations $S_m^{(1)}, S_m^{(2)}$ into a fast converging series involving the exponentially decaying modified Bessel functions K_0 and K_1 (see Appendix)

$$S_m^{(1)} = \frac{2b}{\pi} \sum_{n=-\infty}^{\infty} \left\{ K_0\left(\frac{m\pi}{a}(y - y' + 2nb)\right) - K_0\left(\frac{m\pi}{a}(y + y' + 2nb)\right) \right\} \tag{9}$$

$$S_m^{(2)} = \frac{2ab}{m\pi^2} \sum_{n=-\infty}^{\infty} \left\{ (y - y' + 2nb) K_1\left(\frac{m\pi}{a}(y - y' + 2nb)\right) - (y + y' + 2nb) K_1\left(\frac{m\pi}{a}(y + y' + 2nb)\right) \right\}. \tag{10}$$

Due to the presence of the fast-decaying modified Bessel functions, only two or three terms are required in (9) and (10) for computing $S_m^{(1)}, S_m^{(2)}$. Thus, the original 2-D frequency-independent summations (4)–(6) are effectively transformed into 1-D ones, leading to a dramatic reduction of the associated computational effort. Also, it is important to recognize that since the frequency-independent accelerated expressions (9) and (10) are at the Green's function level, it is implied that any arbitrary discretization scheme can be used for the subsequent implementation of the MoM. The entire procedure can be extended to multilayer substrates in a straightforward way.

III. Moment Method with a Nonuniform Mesh

Based on the Green's function evaluation of the previous section, the MPIE formulation has been applied in a Galerkin's scheme with a nonuniform/nonfixed rectangular mesh based on the rooftop basis functions shown in Fig. 1. This already offers improved flexibility over the fixed mesh required by the FFT schemes. It should be made clear,however that the technique described in Section II enables MoM implementations with any arbitrary kinds of basis functions (such as triangular) for even further geometrical flexibility. Each time, the challenge would be to manage to translate the Green's function speed-up benefits to the particular MoM implementation.

In view of the Green's function decomposition described by (3), the associated MoM impedance matrix can also be decomposed in a similar fashion

$$Z_{\mathrm{MoM}} = Z_D(\omega) + \frac{1}{j\omega}[\hat{Z}^{\mathrm{TM}} - \omega^2 \hat{Z}^{\mathrm{TE}}]. \tag{11}$$

The frequency-dependent component of the MoM impedance matrix $Z_D(\omega)$ needs only a few summation terms to converge due to Kummer's transformation. On the other hand, the frequency-independent components $\hat{Z}^{\mathrm{TE}}, \hat{Z}^{\mathrm{TM}}$ require the evaluation of double integrals of the modified Bessel functions against piecewise constant pulse basis functions. In this case we have managed to reduce *all* frequency-independent interactions in terms of the *single* definite integral of the modified Bessel function of zero order, $K_i(x)$, given in (12)

$$K_i(x) = \int_0^x K_0(u)\, du. \tag{12}$$

The integral (12) is pre-computed numerically, then piecewisely interpolated by polynomials that are stored so that it can be rapidly

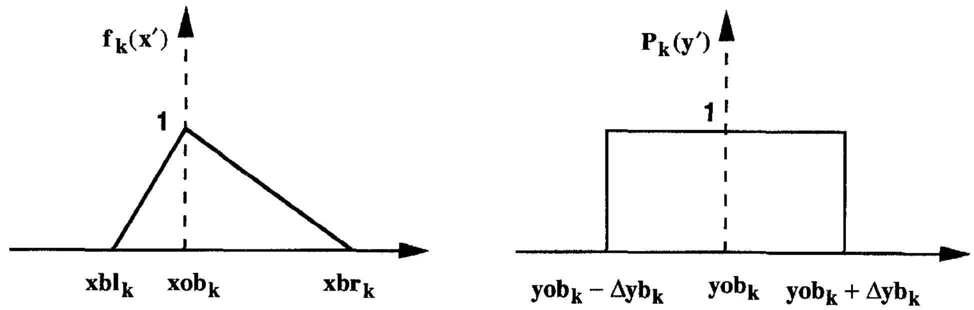

Fig. 1. The nonuniform x-directed rooftop expansion functions $B_k^x(x', y') = f_k(x')P_k(y')$.

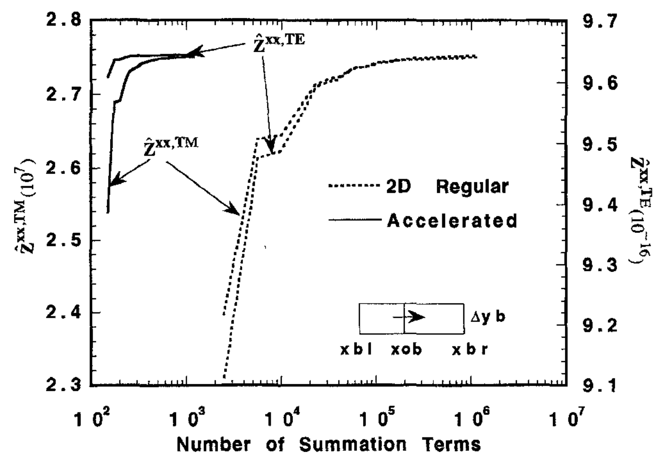

Fig. 2. Convergence of $\hat{Z}^{xx,\text{TE}}$ and $\hat{Z}^{xx,\text{TM}}$. The expansion and testing rooftops coincide at $(xob = 0.3640a, yob = 0.4890b)$ with $xbl = 0.3409a, xbr = 0.4091a$ and $\Delta yb = 0.01136b$. The box size is $a = b = 67.5$ mm, $c = 11.4$ mm and the substrate thickness is $t = 1.57$ mm with $\varepsilon_r = 2.33$.

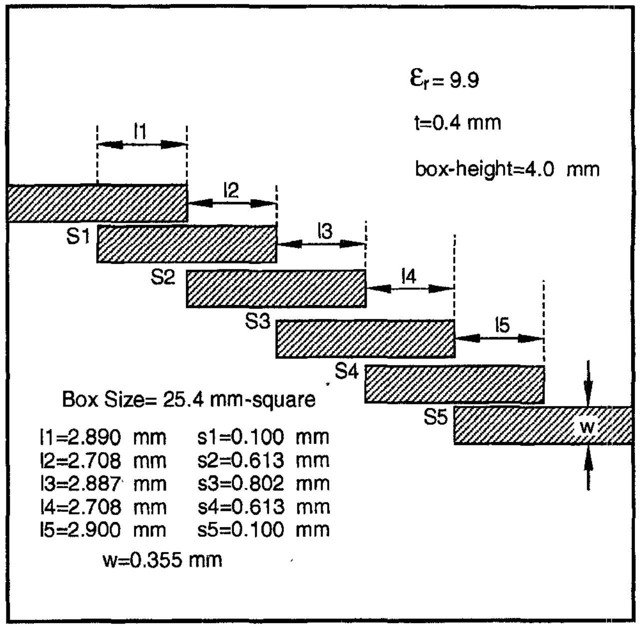

Fig. 3. A bandpass coupled-line microstrip filter on a high-dielectric $\varepsilon_r = 9.9$ substrate.

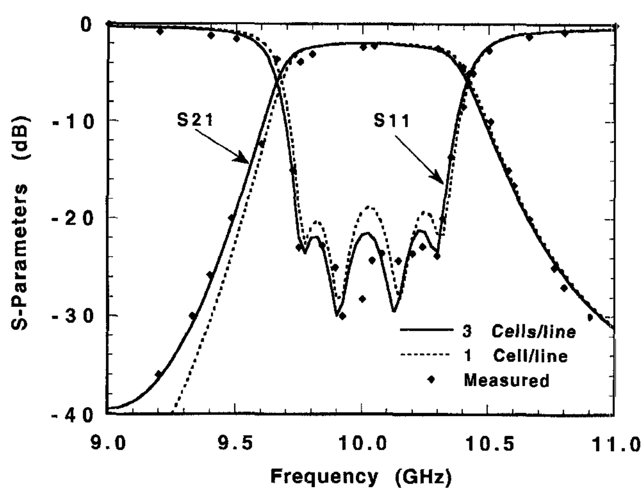

Fig. 4. The measured and computed S-parameters of the coupled-line filter.

retrieved during the frequency-independent MoM computations. To demonstrate the efficiency of the proposed procedure, we show in Fig. 2 the convergence diagrams for the elements of $\hat{Z}^{xx,\text{TE}}$ and $\hat{Z}^{xx,\text{TM}}$ associated to the self-interaction of a rooftop in a typical situation [5]. From Fig. 2., it becomes evident the accelerated formulation reduces the required number of summation terms by two to three orders of magnitude.

IV. Computational and Experimental Results

Consider the coupled-line bandpass filter of Fig. 3, which has quite tight dimensions and is printed on a high-dielectric-constant $\varepsilon_r = 9.9$ substrate. The corresponding simulated return and insertion losses are compared with measurements in Fig. 4. The computations of Fig. 4 have been performed using either one or three transverse cells for each resonator. The frequency-dependent computations are carried out with modes up to $\text{TE}_{60,60}/\text{TM}_{60,60}$. On the other hand, modes up to $\text{TE}_{400,400}/\text{TM}_{400,400}$ are retained for the frequency-independent calculations. The corresponding CPU times on a HP900/735 workstation are 1 h (2 min.) for the initial frequency-independent computations followed by 3.5 min. (11 s) for each frequency calculation when using three transverse cells (one cell) in the microstrip lines. It is important to point out that if the customary technique of carrying out the 2-D summations with the aid of FFT's had been engaged instead, the underlying fixed lattice would have been dictated by the smallest dimension $s_1 = s_5$, which is the spacing between the input and output coupled lines.

V. Conclusion

In this contribution, the groundwork is laid out for the realization of efficient integral-equation/moment-method techniques with arbitrary types of basis functions in a shielded environment. As an example, a

MoM is implemented based on the mixed potential integral equation formulation with a rectangular, but nonuniform and nonfixed, mesh. The entire formulation can be extended to multilayer substrates in a straightforward way.

Appendix

The key equation used is Sommerfeld's identity [11], which expresses a spherical wave in terms of cylindrical ones. This identity is quoted below, although slightly modified to serve our purposes

$$\frac{e^{-jkr}}{r} = \frac{1}{2j}\int_{-\infty}^{\infty} \frac{H_0^{(2)}(\lambda\rho)e^{-jz\sqrt{K^2-\lambda^2}}}{\sqrt{K^2-\lambda^2}}\lambda\, d\lambda, \qquad z>0 \quad \text{(A1)}$$

where the path of integration is along the real-axis but passes above the branch-points at $\lambda = \pm K$ so that the radiation condition be satisfied. By taking the static limit in (A1), $K \to 0$, and using the substitution $\lambda \to j\lambda$, (i.e. transforming the path of integration from the real- to the imaginary axis), the following Fourier-pair relations are readily established:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-j\lambda u} K_0(\lambda\rho)\, d\lambda = \frac{1}{2\sqrt{u^2+\rho^2}} \quad \text{(A2)}$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-j\lambda u}[\lambda K_1(\lambda\rho)]\, d\lambda = \frac{\rho}{2\,{}^{3/2}\sqrt{u^2+\rho^2}}. \quad \text{(A3)}$$

The Fourier pairs (A2) and (A3) are then used together with Poisson's summation formula to convert summations (7) and (8) into their accelerated representations of (9) and (10), respectively.

References

[1] L. P. Dunleavy and P. B. Katehi, "A generalized method for analyzing thin microstrip discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1758–1766, Dec. 1988.

[2] J. C. Rautio and R. F. Harrington, "An electromagnetic time-harmonic analysis of shielded microstrip circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 726–730, Aug. 1987.

[3] A. Hill and V. K. Tripathi, "An efficient algorithm for the three dimensional analysis of passive components and discontinuities for microwave and millimeter wave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 83–91, Jan. 1991.

[4] C. J. Railton and S. A. Meade, "Fast rigorous analysis of shielded planar filters," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 978–985, May 1992.

[5] G. V. Eleftheriades, J. R. Mosig, and M. Guglielmi, "An efficient mixed potential integral equation technique for the analysis of shielded MMIC's," in *Proc. 25th European Microwave Conf.*, Sept. 1995, pp. 825–829.

[6] S. Hashemi-Yeganeh, "On the summation of double infinite series filed computations inside rectangular cavities," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 641–646, Mar. 1995.

[7] J. R. Mosig, "Arbitrarily shaped microstrip structures and their analysis with a mixed potential integral equation," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 314–323, Feb. 1988.

[8] G. V. Eleftheriades and J. R. Mosig, "On the network characterization of planar passive circuits using the method of moments," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 438–445, Mar. 1996.

[9] L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves.* Englewood Cliffs, NJ: Prentice-Hall, pp. 185–217, 1973.

[10] K. Knopp, *Theory and Application of Infinite Series.* New York: Dover, 1990.

[11] J. A. Stratton, *Electromagnetic Theory.* New York: McGraw-Hill, 1941, pp. 576.

Analysis of Electromagnetic Boundary-Value Problems in Inhomogenous Media with the Method of Lines

Arnd Kornatz and Reinhold Pregla

***Abstract*—In this paper we will show how the method of lines can be generalized for the analysis of inhomogenous media. The inhomogeneity is completely arbitrary; the permittivity of the investigated structures may vary in all three coordinate directions. Subjects under investigation are isolated dielectric resonators, mictrostrip filters with dielectric and metallic lossy resonators, and planar capacities.**

I. Introduction

In arbitrary inhomogenous media, electromagnetic fields can only be calculated with numerical methods. Possible methods are mode-matching methods [1], finite element methods [2], or finite difference methods [3]. As long as the medium is structured in some way, the numerical analysis is partly substitutable by analytical calculations. A method that is based on this principle is the method of lines [4]. If the structure is invariant in one coordinate direction, the fields can be calculated analytically in this direction. In the other directions, the calculation is furthermore discrete. In comparison to the above-mentioned methods [1]–[3] this procedure needs less computational resources. Under the use of Cartesian coordinates, the method of lines can be employed to analyze all structures that consist of layers in which the material does not change in normal direction. Every structure can be separated in such layers so that the method of lines is an universal tool for the analysis of arbitrary microwave components. Simple examples for layered structures are microwave filters with dielectric or metallic resonators, planar capacitors, optical modulators, or couplers. In spite of the differences between these structures (e.g., used materials, boundary conditions, and ranges of application), they can be analyzed with the same theory.

II. Theory

A. Electrodynamic Applications

For the analysis of inhomogenous layers it is assumed that the permittivity ε_r varies in x and y-direction, but not in z-direction. In this case the electromagnetic field can be derived from a vector potential $\boldsymbol{A}$. It is important that the potential has the same vector components as the gradient of the permittivity

$$\boldsymbol{A} = A_x \cdot \boldsymbol{e}_x + A_y \cdot \boldsymbol{e}_y. \quad (1)$$

Only this general solution leads to a consistent system of coupled differential equations for the potential components A_x and A_y

$$\varepsilon_r \frac{\partial}{\partial x}\left[\frac{1}{\varepsilon_r}\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y}\right)\right] + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} + k_0^2\varepsilon_r A_x = \frac{\partial}{\partial y}\left(\frac{\partial A_y}{\partial x}\right) \quad (2)$$

$$\frac{\partial^2 A_y}{\partial x^2} + \varepsilon_r \frac{\partial}{\partial y}\left[\frac{1}{\varepsilon_r}\left(\frac{\partial A_y}{\partial y} + \frac{\partial A_x}{\partial x}\right)\right] + \frac{\partial^2 A_y}{\partial z^2} + k_0^2\varepsilon_r A_y = \frac{\partial}{\partial x}\left(\frac{\partial A_x}{\partial y}\right). \quad (3)$$

Manuscript received November 10, 1995; revised August 26, 1996.

The authors are with the Allgemeine und Theoretische Elektrotechnik, Fern Universitaet Hagen, D-58084 Hagen, Germany.

Publisher Item Identifier S 0018-9480(96)08504-3.